CHAPTER 1

A Vision for Learning Mathematics

Three documents have been a driving force in the change of mathematics teaching in the United States and in Canada. For educators of mathematics in North America, the release of the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics* in 1989 provided a clear vision. This was a historic move by a professional organization that articulated extensive goals for teachers and policymakers in mathematics. In 1991, NCTM published *Professional Standards for Teaching Mathematics,* which described the elements of effective mathematics teaching. In 1995, *Assessment Standards for School Mathematics* was published. In 2000, *Principles and Standards for School Mathematics* followed: This document was a review and revision of the above three documents and is one resource that can be used to help improve mathematics teaching, assessment, and curriculum (NCTM, 2000).

The mathematics that one needs to know has shifted. Technology has been a major factor in changing our homes, workplaces, and daily lives. As new technological applications emerge, new mathematics is being created. Has the mathematics teaching and learning in your school changed, or has it remained relatively the same as when you went to school? Do the mathematics

programs in your schools still rely heavily on traditional procedures, such as "Yours is not to reason why, just invert and multiply?" Or are teachers and students engaged in mathematics for understanding? Mathematical power can, and must, be at the command of all students in a technological society. Mathematics is something one participates in and does, sees, hears, and touches in meaningful ways. It has broad content encompassing many fields. It is imperative that mathematics teaching and learning in schools is changed to reflect the current research and the changes in society in general.

The vision of mathematics today includes beliefs that students should learn to value mathematics. This new vision espouses that the teaching of mathematics well calls for increasing our understanding of the mathematics we teach, seeking greater insight into how children learn mathematics, and refining lessons to best promote children's learning. Students should be able to reason and communicate mathematically. Problem solving is a life skill that students will need to be successful in their daily lives and in their chosen careers. In today's mathematics classrooms, students and teachers should be part of a mathematical community working together to solve problems rather than only being involved in independent work.

Students need to use logic and mathematical evidence to provide verification for correct answers, rather than the teacher being the sole authority in mathematics. Mathematical reasoning needs to become more important than the memorization of procedures. The overall objective in teaching mathematics is to develop, in each and every student, an understanding and love of mathematics that lasts a lifetime and evolves to meet changing demands. An effective mathematics program should focus on conjecturing, inventing, and problem solving rather than merely finding correct answers. Frequent discussions in the form of individual and small- and large-group conferences; debriefings; or mathematical congresses about problem-solving strategies, mathematical processes, and solutions are important. It is imperative that we move away from treating mathematics as a body of isolated skills and that we present mathematics by connecting its ideas and applications, particularly focusing on the connections of mathematics and the students' real lives.

A clear vision for learning mathematics is one where students engage in meaningful mathematics experiences through the use of concrete materials and manipulatives, visuals, technology, and other resources (see Chapter 4: "Tools for Success"). It is important for

students to build on their prior learning and knowledge of key math concepts and to make connections to their own world. Inquiry, problem solving, discussion, and question posing are all important parts of mathematics learning. Teachers need to use a variety of learning and assessment strategies to accommodate the different learning styles of their students. Mathematics investigations are important for students to engage in, as they can provide multiple opportunities for students to learn and apply mathematics in creative and purposeful ways. It is important for teachers to provide meaningful feedback (formative assessment or assessment for learning) to their students throughout the mathematics learning process. This is important for student improvement and for teacher reflection on the effectiveness of the mathematics opportunities they are providing for their students. It is important for teachers and administrators to engage in ongoing professional growth opportunities and for them to reflect continuously on the mathematics teaching and learning that is happening in their classrooms and schools. This can be done most effectively through job-embedded learning techniques, such as professional learning communities or teams (PLCs or PLTs); peer coaching; group lesson study; and collaborative planning, scoring, and marking. All members of the school community, including educators, students, and parents, should be actively involved in meaningful mathematics (see Resource 1: Ontario Association for Mathematics Education Vision Statement).

MATHEMATICS REFORM

In the early 1980s, educators were faced with a cry for a "back to basics" mathematics curriculum, which was a reaction to the "new math" of the 1960s and 1970s. At the same time, there was increasing interest in problem solving as a focus of mathematics education. As a result of the research of Piaget and other developmental psychologists, mathematics educators were shifting the focus from content to how children can best learn mathematics. Mathematics curriculum, pedagogy, and epistemology have undergone intense rethinking in the past decade and a half. A great responsibility for the success of reform has been placed on the classroom teacher, according to recent mathematics education documents (NCTM, 1991; NRC, 1989; Romberg & Carpenter, 1986).

The role of the teacher has shifted from expert "information/answer giver" to guide, facilitator, listener, and observer. The

emphasis in the classroom has moved from traditional, skill-based procedural tasks to problem solving and reasoning. There is now cross-strand, cross-category, and cross-subject integration of mathematics tasks. There is communication and discourse about mathematics topics and increased use of technology, manipulatives, and group work. Students are involved with contextual, real-life problems that focus on developing problem-solving strategies rather than finding one single correct answer. Teaching for meaning and understanding is the goal, with the use of a variety of strategies to help students visualize abstract ideas (e.g., pictures, graphs, models, technology, language). The mathematics program also includes authentic, complex, multidimensional assessments. To some classroom teachers, many of these ideas may be new and, therefore, may require changes in practice to make them a reality.

Implementing educational reforms for teaching and learning places profound demands on teachers. If teachers are to move toward these reform visions, all teachers (novices and experts) will need to make major changes in their knowledge and beliefs about mathematics learning and teaching, as well as in their teaching practices. The changes teachers are expected to make require large amounts of time and professional development support. Many teachers' images and beliefs about mathematics and what mathematics learning involves may still be incompatible with current research and reform efforts in the field.

Several factors have influenced mathematics reform. The National Council of Teachers of Mathematics has been the main driving force in the current mathematics reform movement in North America. Other factors include national and international assessments that compare student performance, whose results appear regularly in the news media. For example, in many states, the state test results are published in local newspapers with rankings from lowest to highest. This creates a huge stigma for schools, particularly if the contextual data about mobility and socioeconomic and demographic information are not published along with the test scores. As well, curriculum documents and commercial textbooks are major factors in the reform movement. Unfortunately, many teachers have not been given the appropriate professional development to understand the philosophy and pedagogy behind the reform-based textbooks. As a result, many teachers may be using these textbooks in a more traditional

manner (for example, open the text book to the next page, teach the lesson, assign the questions, assign homework, take up the homework at the beginning of the next day's math class), which is a less effective way of teaching. This approach also renders the new textbooks a poor investment, as they are not being used for their intended purpose (for example, the new textbooks are meant to enhance problem-solving skills and promote deep conceptual understanding through the three-part lesson model: explore the problem in a group or with a partner using a student-generated strategy, connect via a teacher lesson related to the problem and what was observed while students were solving the problem, and a reflection or math debrief/congress where the group discusses effective strategies and understanding of the concepts).

TEACHER CHANGE IN MATHEMATICS

As a school administrator supporting your school community in its mathematics instruction and assessment initiatives, it is important for you to be informed of current research in mathematics education.

The school administrators as well as the teachers must have some understanding of the change process in education. Research has shown us that the one-shot, workshop-based professional development opportunities that teachers have usually been offered may result in some improved mathematics in individual classrooms. However, necessary long-lasting, schoolwide change calls for substantive, ongoing, schoolwide support with continuous formal and informal job-embedded learning strategies being used with your school faculty.

For real change to happen, schools need to plan carefully. According to Michael Fullan and Andy Hargreaves (1992), there are three parts to the change process:

1. *Providing new materials.* This is the tangible part of a change innovation and relatively easy to accomplish.
2. *Introducing new behaviors and practices, or the "doing" part of the change process.* This is introducing and supporting the different pedagogical style, skills, and practices in which a person will be involved. This would involve changing a teacher's mathematics instruction from a traditional

skill-based program to a more balanced, reform-based program based on rich learning tasks, problem solving, and deep conceptual understanding.

3. *Embedding new beliefs and understandings.* This is what makes innovation happen—where one internalizes and understands the rationale for the change. This understanding is very important when making the decision about whether or not to implement the change and how to use it. This third part is the crux of the change process in mathematics education. It is absolutely imperative that school administrators, teachers, and the school community have an understanding of the reform movement in mathematics and the change process itself, as both are essential for any meaningful change in mathematics teaching and assessment to happen.

The decades of the 1990s and the first decade of the new millennium have had the ongoing theme of "large-scale reform." We have seen those testing initiatives change in a variety of ways over the past decade. The data sometimes show improvement, but they never seem to show enough improvement. The question is, how do we do better at closing the achievement gap? What qualities do our educational leaders and faculty need to best drive the improvement agenda for a sustainable period of time?

"Sustainability is the capacity of a system to engage in the complexities of continuous improvement consistent with deep values of human purpose" (Fullan, 2005). Fullan goes on to explain that sustainability isn't just about ensuring that an initiative lasts a long time; it also addresses the fact that new initiatives may need to be developed without compromising existing initiatives.

As a school administrator, you face the challenge of deciding which initiatives are worthwhile and necessary for a school community and then moving the school in that direction. Once you have decided, together with your staff and school community, that mathematics is a focus for your change initiative, it has been shown that the success or failure of an education program is determined more by whether it upholds or challenges the school community's beliefs than by the number or breadth of the changes involved in the initiative.

As stated above, teachers' values, beliefs, and reflections are important in the change process. Several elements have been

observed in studies of successful educational reform: shared leadership, awareness of need, adult interaction with each other, ongoing commitment in spite of conflict and tension, desire to learn, and parental support. Elizabeth Smith Senger (1999), in her study of three elementary teachers who were struggling with issues of reform and traditional mathematics teaching in relation to their personal values and beliefs, found some observed paths of teacher change:

> The first, newly gained awareness, was initially held without commitment in a tentative questioning mode and was pooled from various sources. Essentially, the teacher has simply gained an awareness of the need for change in his or her teaching; however, no change or commitment to change has been made at this point. If the teacher did not reject this new information and awareness, with time and reflection, produced mental images of new forms of teaching practice (pre-images) are formed. The teacher is thinking about new or different ways to teach and is beginning to visualize them. The pre-images inspired a double-faceted experimentation which involved both verbal and classroom practice trials. The teacher willingly begins to experiment with teaching methods as a result of the visualizations. Verbal experimentation involved the teachers using expressions and descriptions of their pre-images as a means of expressing and assessing several aspects which included:
>
> (a) Their own comfort level
>
> (b) Their confidence that the ideas would work in their classrooms
>
> (c) The ways in which the ideas "fit" with the teacher's own past history
>
> (d) The teacher's reputation with her colleagues as "traditional" or "reform-oriented" in terms of initiative and mathematics teaching. (Senger, pp. 210–211)

Senger's data revealed that the integration of a new belief occurred as a thoughtful and complex process over time, which included imaging, experimentation, and reflection on those values. One complication for teachers when attempting to implement change

in the area of mathematics is that, unlike teaching other subjects such as literacy or history, most teachers face the extra challenge of not having a solid understanding of the mathematics content itself.

Borko, Mayfield, Marion, Flexer, and Cumbo (1997), in a study of a group of third-grade teachers who participated in a University of Colorado assessment project on designing and implementing mathematics performance assessment tasks, found the following supports and impediments to teacher change:

> *Theme 1:* Situating the change process in the actual teaching and learning contexts where the new ideas will be implemented is an effective strategy in helping teachers change their practice. This is job-embedded learning in the teachers' own classroom and school with their own colleagues.
>
> *Theme 2:* Group discussion of instructional and assessment issues can be an effective tool for the social construction of new ideas and practices. Just as students need to have math conferences or a math congress, so do teachers. This discussion and sharing of practice helps teachers solidify their understanding of the mathematics being taught and their knowledge and understanding of what and how their students are learning. It also helps build confidence so that teachers are willing to continue with the change process.
>
> *Theme 3:* Staff development personnel and other persons with specific expertise can facilitate change by introducing new ideas based on teachers' current levels of interest, understanding, and skill.
>
> *Theme 4:* When teachers' beliefs are incompatible with the intentions of the staff development team and are not challenged, the teachers are likely either to ignore new ideas or inappropriately assimilate them into their existing practices.
>
> *Theme 5:* Time is a major obstacle to changing classroom practice. Competition among priorities for limited classroom time is particularly troublesome, as well as for time for necessary job-embedded staff development that occurs during the school day. (pp. 14–26)

Researchers also acknowledged that teachers' knowledge and beliefs about learning, teaching, and mathematics subject matter are critical in determining how and whether teachers implement new educational ideas. If teachers believe in traditional teaching

(put simply, the teaching of isolated rote skills out of context and not connected to real-world problems) and ignore the research on mathematics reform and on how students construct learning in mathematics, then it is highly unlikely that teachers will change their practice.

Another major factor in the change process is trust. For trust to be developed in a school, both teachers and leaders need to have discovered that it is safe to take risks and chances and that it is safe to make mistakes. The staff need to feel that they are working in a protected learning environment where mistakes will be made but that they will be learned from and that the small successes will be celebrated along the way.

McLaughlin and Talbert (1993) suggested that, for teachers to adapt their teaching practices to meet the new reform agenda, they must participate in a professional learning community that supports risk taking and that discusses new teaching strategies and materials. Borko et al. (1997) also discuss the issue of time in educational reform. They suggest that staff development programs be at least one year in length and that they provide release time for teachers. Change efforts must take into account the feeling that classroom time is insufficient for teachers to accomplish what they feel they need to with the students.

TEACHERS' ATTITUDES AND PERCEPTIONS ABOUT MATHEMATICS TEACHING AND LEARNING

Teachers' own beliefs about mathematics, how children learn mathematics, and what constitutes good teaching affect the way teachers choose to teach mathematics and what they choose to teach.

Research has shown us that many graduates of teacher education programs still end up teaching the way they were taught as students (usually in a traditional manner) despite the quality of their teacher education program. This must change in light of our changing society and the current research on effective mathematics teaching and learning. It is a major challenge for school administrators if a teacher does not believe that change is necessary in the mathematics program.

Teacher beliefs, and the choices teachers make, can have a major impact on how students view mathematics and their learning

of it. According to research, it is suggested that teachers' beliefs about mathematics are often limited and may be dualistic, in the sense of having a traditional right/wrong orientation and using mostly single procedures to arrive at the correct answer. A consistent theme found in Cooney, Wilson, Albright, and Chauvots (1998) RADIATE study was that teachers equated good teaching with good telling. In other words, students should understand mathematics step by step and should not be confused. A second theme that was found was that of "caring." Because teachers cared about their students' success in mathematics class, they felt that caring meant enabling students to master basic skills, often putting aside challenging tasks on assessments for those that mimicked the traditional skill-based lessons done in class. This is a reductionist orientation that is counter to reform efforts in mathematics.

Baroody (1998) provides a summary of research on three different views of mathematics that have been identified among teachers:

1. Mathematics as a collection of unrelated basic skills
2. Mathematics as a coherent network of skills and concepts (mathematics as a static body of knowledge)
3. Mathematics as a way of thinking (inquiry process, mathematics as a dynamic field)

Knowledge and beliefs are inextricably intertwined. Our beliefs are like a filter through which new phenomena are interpreted. A teacher's sense of purpose as a mathematics teacher, philosophy of learning and teaching, and sense of responsibility in terms of the community in which he or she teaches are all fused with what the teacher "knows." As well, it is important for teachers to be reflective practitioners. In the case of mathematics, teachers need to see mathematics as a creation of knowledge rooted in rationality. Mathematics knowledge is not static; it is fluid. Context and reflection play an important role in allowing the knowledge required by reform to be fluid and flexible. Both "what the teacher knows" and the way the knowledge is acquired are important issues.

Administrators can push teachers to change their classroom activities, but we also need to change their fundamental beliefs and attitudes about teaching and learning, the roles of teachers and students, and how teaching and learning should be carried

out. For change to be successful, teachers' beliefs, attitudes, and practices need to be aligned.

It seems logical that influencing teachers' beliefs may be essential to changing teachers' classroom practices. At one end of the beliefs continuum are traditional beliefs. Stipek, Givvin, Salmon, and MacGyvers (2001) found that teachers who scored high on these more traditional beliefs were less self-confident about teaching mathematics and enjoyed it less. In their data analysis, five dimensions of beliefs (more traditional beliefs linked to teachers' being less confident about teaching mathematics) were strongly associated with each other:

1. Mathematics is a set of operations to be learned.
2. Students' goal is to get correct solutions.
3. The teacher needs to exercise complete control over mathematics activities.
4. Mathematics ability is fixed and stable.
5. Extrinsic rewards and grades are effective strategies for motivating students to engage in mathematics.

If one looks at the opposite end of the dimensions (reform-based beliefs linked to teachers' being more confident about teaching mathematics), there was consistency in the following beliefs:

1. Mathematics is a tool for thought.
2. Students' goal is to understand.
3. Students should have some autonomy.
4. Mathematics ability is amenable to change.
5. Students will want to engage in mathematics tasks if the tasks are interesting and challenging (not for extrinsic rewards).

The authors speculate that building teachers' self-confidence in mathematics (which requires building their mathematical understanding) could be an important and perhaps necessary criterion in moving teachers toward more inquiry-oriented beliefs and practices. If this suggestion is valid, the school administrator's challenge in this

area is to be able to provide the intensive, sustainable professional development required to improve teacher understanding of mathematics, to improve confidence in the subject area, and to change beliefs and attitudes about teaching and learning mathematics.

TEACHERS' UNDERSTANDING OF MATHEMATICS CONTENT AND PEDAGOGY

Many teachers and school administrators state that they are not comfortable teaching mathematics. They remark that they don't have a good mathematics background; they don't understand mathematics; and, in many cases, they admit that they are "math phobic." Any school administrator who has participated in parent-teacher interviews has regularly heard parents proudly state, "I can't do math. I'm no good at math and never have been." Parents, and sometimes teachers, can be heard making such statements in front of their children. As an individual in our society, one would never proudly admit that one couldn't read or that one was illiterate. Why is it that it is seen as acceptable to admit that we cannot do math or that we are not numerate? John Van De Walle (2003) states in his book *Elementary and Middle School Mathematics: Teaching Developmentally,* "For teachers of mathematics to be truly effective involves bringing together four basic components:

1. an appreciation of the discipline of mathematics itself—what it means to 'do mathematics'
2. an understanding of how students learn and construct ideas
3. an ability to design and select tasks so that students learn mathematics in a problem-solving environment
4. the ability to integrate assessment with the teaching process in order to enhance learning and improve daily instruction." (p. 1)

In line with the NCTM *Principles and Standards* (2000), there is a clear differentiation between knowledge *of* mathematics (the subject matter) and knowledge *about* mathematics (its nature and discourse). Both are necessary for effective mathematics teaching (Ball, 1991). Knowledge *about* mathematics includes, according to Ball:

> Understanding about the nature of mathematical knowledge and activity: what is entailed in doing mathematics and how truth is established in the domain. What counts as a solution in mathematics? How are solutions justified and conjectures disproved? Which ideas are arbitrary or conventional and which are necessary or logical? Knowledge about mathematics entails understanding the role of mathematical tools and accepted knowledge in the pursuit of new ideas, generalizations, and procedures. (p. 7)

It is important to note that knowing *about* mathematics, in the manner that Ball (1991) describes, is substantially different than knowledge *of* mathematics.

Knowledge *of* mathematics constitutes an awareness of mathematics as a discipline, characterized by specific concepts, procedures, and accurate results—a view that leads to an instructional focus on practice and drill and has dominated school mathematics and testing programs up to the present time.

Mathematical knowledge alone does not translate into better teaching. According to Lappan and Theule-Lubienski (1994), teachers need at least three kinds of knowledge to be somewhat effective in choosing worthwhile tasks:

1. Orchestrating discourse (for example, mathematics conferences, debriefs, or congresses)
2. Creating an environment for learning
3. Analyzing their teaching and student learning

In the case of mathematics, this translates into knowledge about mathematics, knowledge about students, and knowledge about mathematics pedagogy. Teachers often lack fundamental understandings of school mathematics, even though they may have excelled in advanced-level university mathematics courses. Philosophy and cognitive psychology support the view that what teachers learn is framed in the context in which that knowledge is acquired. In other words, we often teach mathematics the way we were taught it. Many practicing teachers' views of mathematics teaching are consistent with the way they experienced mathematics teaching, which is usually counter to current mathematics reform initiatives.

How do teachers and school administrators move beyond the lack of mathematical understanding if they don't take courses about elementary school mathematics (content and pedagogy)? How can teachers have a deeper understanding about the mathematics they are teaching, particularly if they have not engaged in quality professional development programs in mathematics education? Teachers need to be involved in ongoing job-embedded staff development that revolves around the mathematics teaching and learning happening in their own classrooms. As the school leader, you need to be able to facilitate the time and the discourse in these sessions for improvement in the teaching and learning of mathematics for both teachers and students. More detailed job-embedded learning strategies will be outlined in upcoming chapters.

THE PRINCIPAL'S ROLE

The principal's role in leading the school toward the current vision of mathematics education is to facilitate a culture of improvement. The subsequent chapters in this book will provide the school leader with strategies and tools to develop that culture and to support teachers, students, and parents in moving through the mathematics improvement initiatives.

Chapter 2 will provide strategies for identifying teacher practices in exemplary mathematics classrooms, including a template for observing and evaluating a mathematics classroom. This tool will assist with defining criteria for classroom walk-throughs and teacher performance appraisals. The chapter will also provide an overview of the use of problem solving as the main driver for the mathematics program. The effective use of mathematics manipulatives will be discussed and lists of mathematics manipulatives for different grade levels provided. There is also a section on technology and how it is used effectively in the reform-based mathematics classroom.

The section on literature connections and how children's literature can enhance the mathematics program is included with reference to lists of mathematics literature. A comprehensive section on assessment and evaluation will assist the school principal in supporting teachers in providing balanced mathematic instructional and assessment criteria in the mathematics program.