Estimating Quotients

Determine the best estimate.	Explain your choice.
1. **22.5 ÷ 0.54** a. Between 4 and 5 b. Between 40 and 50 c. Between 400 and 500	
2. **0.683 ÷ 1.9** a. Between 0.1 and 0.5 b. Between 10 and 50 c. Between 100 and 500	
3. **8.4 ÷ 0.04** a. Between 0.2 and 0.4 b. Between 20 and 40 c. Between 200 and 400	

Teacher Notes: Estimating Quotients

Questions to Consider About the Key Mathematical Concepts

When solving problems involving the division of decimals, can students reason about the size of the numbers and the effect of the operation to determine a reasonable estimate? To what extent do they

- reason correctly about the size of the divisor and dividend?
- determine whether the quotient will be smaller or larger than the dividend?
- describe how to use this reasoning to determine an estimate?

Common Core Connection (CCSS.Math.Content.6.NS.B.2 and CCSS.Math. Content.6.NS.B.3)

Grade: Sixth

Domain: The Number System

Cluster:

Compute fluently with multi-digit numbers and find common factors and multiples.

2. Fluently divide multi-digit numbers using the standard algorithm.
3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Uncovering Student Understanding About the Key Concepts

Using the Estimating Quotients Probe can provide the following information about how the students are thinking about the effect of operating with decimals.

Do they		*Do they*
• correctly reason about the size of the dividend and divisor?	OR	• apply incorrect place value thinking?
• correctly reason about the size of the quotient?	OR	• apply an overgeneralization of "division always results in a smaller answer"?
• use reasoning about the size of the decimals and the effect of the division?	OR	• revert to applying an algorithm rather than using reasoning to determine an estimate?

Exploring Excerpts From Educational Resources and Related Research

Common areas of difficulty for students:

> Multiplying and dividing fractions and decimals can be challenging for many students because of problems that are primarily conceptual rather than procedural. From their experience with whole numbers, many students appear to develop a belief that "multiplication makes bigger and division make smaller." (NCTM, 2000, p. 218)

> Errors show that many students have learned rules for manipulating symbols without understanding what those symbols mean or why the rules work. Many students are unable to reason appropriately about symbols for rational numbers and do not have the strategic competence that would allow them to catch their mistakes. (NRC, 2001, p. 234)

> Developing fluency requires a balance and connection between conceptual understanding and computational proficiency. Computational methods that are overpracticed without understanding are often forgotten or remembered incorrectly (Hiebert 1999; Kamii, Lewis, and Livingston 1993; Hiebert and Lindquist 1990). (NCTM, 2000, p. 35)

Surveying the Prompts and Selected Responses in the Probe

The Probe consists of three separate selected response items. The prompts and selected responses are designed to elicit understandings and common difficulties as described the following table.

If a student chooses	*It is likely that the student*
1b, 2a, 3c (correct answers)	• is correctly reasoning about the size of the decimals and the effect of the operation [See Sample Student Response 1]; or • has correctly applied an algorithm rather than using reasoning to determine an estimate [See Sample Student Response 2]. *Look for indication of the student's understanding in the written explanations of how the student got the answer.*
1a, 3a	• is applying the incorrect rule of "division makes smaller." Note that applying this rule results in selecting the correct response, 2a, for #2 [See Sample Student Response 3].
Various other patterns	• inconsistently applies correct reasoning about either the size of the decimal or the effect of the operation; and/or • has incorrectly applied an algorithm [See Sample Student Responses 4 and 5].

Teaching Implications and Considerations

Ideas for eliciting more information from students about their understanding and difficulties:

- For students who incorrectly reason about the size of the decimals, ask, "What benchmark or whole number is this number close to?"
- For students who apply the "division makes smaller" rule, ask, "How can you model 2 divided by 0.5?" If they are able to correctly show that there are 4 "halves" in 2, follow up with, "How can this help you think about the problems?"
- For students who apply an algorithm rather than estimating, ask, "How might you think about this without actually calculating an answer?"

Ideas for planning instruction in response to what you learned from the results of administering the Probe:

- Use representations and concrete models such as number lines and base-ten blocks to help students' understand division as how many ___ in ___?
- Provide a problem context from which students can make sense of the results.

- Focus on the meaning of the operation before introducing steps of an algorithm.
- Continue to require students to reason about the size of the numbers and the effect of the operation to determine an estimate as a method of checking for the reasonableness of the results of applying an algorithm.

Sample Student Responses to Estimating Quotients

Responses That Suggest Understanding

Sample Student Response 1

Probe Item 1. b. If you have 20 and you want to know how many .5's go into 20, it will double it. Takes two .5 for every 1. That means it is about 40.

Probe Item 2. a. If you have about .5 and want to know how many 2's go into .5 then it can't even be 1.

Probe Item 3. c. If you have about 8 and divide into really small amounts like pennies then it would be a lot. 4 pennies in $1 is 25. 8 of these would be 200.

Sample Student Response 2

Probe Item 1. b. I rounded .54 to 50 and moved the decimal. $50\overline{)2250}^{\,45}$. 45 is between 40 and 50.

Responses That Suggest Difficulty

Sample Student Response 3

Probe Item 3. b. I am thinking between 20 and 40 makes sense. 8 ÷ 4 is 2 but you are talking about decimals here.

Sample Student Response 4

Probe Item 2. a. 0.683 rounds to 1 and 1.9 rounds to 2. 1 ÷ 2 is 2. None of the answers work but since I rounded I will pick "a" since it is closest to 2.

Sample Student Response 5

Probe Item 3. b. Estimate then divide, move the decimals to divide. $4\overline{)80}^{\,20}$.