

Chapter 4

Model With Mathematics

ALIGNING CHAPTER 4 TO THE COMMON CORE STATE STANDARDS



Applying mathematics to everyday life brings the classroom alive! The fourth of the Common Core State Standards for Mathematics (CCSSM) Standards for Mathematical Practice calls for students to “Model with mathematics.” From the use of visual representations to traditional school activities such as fundraising and games, Chapter 4 includes research and ideas for incorporating real-life examples and inquiry-based activities into the mathematics classroom.

Strategy 22: Find out about your students’ motivation regarding mathematics, and use that knowledge to refine your instruction.



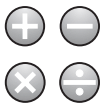
What the Research Says

A study examining motivation in the mathematics classroom focused on how teachers tried to build motivation into their lessons and how teachers’ motivational beliefs compared with students’ motivational beliefs. Students and teachers were asked to fill out a grid in which they identified characteristics that they believed made mathematics interesting. Teachers and students



identified the same characteristics. They both emphasized the relationship between arousal (what stimulated students' interest) and control levels (what directed their problem-solving activities). The results showed that teachers had little knowledge about what made mathematics motivating to their students. Findings from this study suggest that

- teachers need to learn what makes mathematics interesting to students,
- teachers need to pay attention to individual differences in student motivation, and
- when teachers know about their students' motivational beliefs, they are more capable of refining their instruction so that students are interested in mathematics.



Teaching to the National Council of Teachers of Mathematics Standards

The National Council of Teachers of Mathematics Teaching Principle states that “effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.”¹ Since teachers are living their lives in a different universe from students, it is highly recommended that teachers do everything to learn about their students' interests. When teachers know what interests students, they can plan lessons that employ that knowledge. In many instances psychological motivation can also be effective. Teachers can motivate students by imploring them to learn a little more so that they can complete a topic or chapter. Furthermore, teachers can motivate students by highlighting that mastery of mathematics is essential for success in other subjects and for success on important standardized tests for college admission.

The National Council of Teachers of Mathematics Connections Standard calls for students to “recognize and apply mathematics in contexts outside the classroom.”² You may wish to regularly ask students to write about the relevance of each lesson to their daily lives. You will be delighted to learn of the “surprise” connections that students make with mathematics. As you prepare lessons in the future, you will have a rich assortment of “connections” that can motivate students.

Aligning to the Common Core State Standards



Opening the year with a “Student Interest Survey” will provide you with an inventory of what appeals to your students. Use every opportunity to incorporate student interest in the problems that you select, and when there are no interesting problems to select based on your students’ priorities, then make them up! In my algebra classroom in an urban high school, we listened to selections of hit music and timed how fast each artist rapped and then charted the results using direct variation to compare their rates. Students, years later, recall that lesson fondly. In addition, students who were at times unengaged were very present in that particular lesson. I also used earnings from weekend movies to review scientific notation and scores from baseball games to review ratios. Students who see mathematics connected with their world are more engaged in the material.

Classroom Applications



The concept of intrinsic motivation means using interests already present in the learner to generate motivation or excitement in the subject matter. From this very definition, it is implicit that the teacher develops sensitivity for what interests students. This can vary from region to region and from student to student, and may vary with age, gender, and cultural background as well. There are, however, some relatively universal factors that many people harbor as interests, for example, the concept of completion. Students have the desire to feel that they have completed a task or topic and have relatively complete command of a concept. When teachers can craftily have students realize that their mastery is not yet complete, but with a little bit of further study it can be complete, they will have used a classic technique for motivating many students. A more complete treatment of this motivational technique can be found in Posamentier, Smith, and Stepelman (2010).

Precautions and Possible Pitfalls



Teachers need to cultivate the ability to determine what really motivates their specific students instead of assuming students are motivated by the same things that motivate the teachers themselves. To motivate students effectively, problems and topics must be appropriate in their content, structure, and level of difficulty.

Sources

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Strategy 23: When trying to determine how to motivate students' interest in mathematics, teachers should differentiate between personal and situational interest and use both forms to increase students' motivation to learn mathematics. Teachers also need to both stimulate and maintain their students' interest.

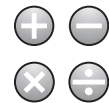


What the Research Says

Teachers can draw on different types of interest that students have in mathematics. Personal interest is what students bring with them to the classroom or other environment; situational interest is something that is acquired by participating in an activity in the classroom or another situation. Whereas personal interest emphasizes the importance of working with individual differences in motivation, situational interest emphasizes the importance of the teacher's creating an appropriate setting to develop the students' interest in mathematics. Teachers should also differentiate between factors that stimulate student interest and those that maintain student interest. Computers, puzzles, and group work tend to stimulate interest in mathematics while meaningfulness and involvement tend to maintain student interest.

In a study of 350 high school students from three high schools, students were administered an interest survey with seven scales: personal interest, situational interest, meaningfulness, involvement, puzzles, computers, and group work. Students rated items on a six-point scale ranging from *strongly disagree* to *strongly agree*. Most students were white and were in 13 college preparatory algebra or geometry classes. The results showed that situational interest in secondary school mathematics classrooms is complex, having five different components: meaningfulness, puzzles, computers, group work, and involvement. Increasing student involvement in mathematics appears to be especially beneficial for enhancing situational interest.

Teaching to the National Council of Teachers of Mathematics Standards



The National Council of Teachers of Mathematics Connections Standard states that students should “recognize and apply mathematics in contexts outside of mathematics.”³ Student learning is most effective when students are motivated. When teachers can incorporate personal interests of students in the mathematics that is being taught, students can better appreciate mathematics because it plays a role in their daily lives. When presenting topics that are not among students’ interests, teachers must create or manufacture a “situational” motivating component. Each of the eight techniques listed in the following can effectively motivate students to learn a lesson.

Aligning to the Common Core State Standards



Mathematical modeling can be approached in a variety of ways. Understanding that personal and situational interest can both be leveraged to create engaging modeling problems will allow educators to enhance the variety of problems they pose to their students. The research points to creative ways to use student interest in creating class activities, so be sure to use the ideas in this chapter to fill your units with modeling challenges for your students!

Classroom Applications

There are many techniques for creating a situational motivation in the classroom. Sometimes the motivation lies in the material and



other times it is dependent on the manner in which the activity is presented. Here are eight techniques for motivating a lesson:

1. Indicate a void in the students' knowledge.
2. Present a challenge.
3. Show sequential achievement.
4. Indicate the usefulness of a topic.
5. Use recreational mathematics.
6. Tell a pertinent story.
7. Get students actively involved in justifying mathematical curiosities.
8. Use teacher-made or commercially prepared materials or devices.

For example, when motivating students about a topic (or lesson) on digit problems in the algebra class or when the class is beginning to understand the workings of the decimal system (algebraically), you might have students try to explain the mathematical novelty:

Why does the following arithmetic always result in the same number, 1089?

Do the following:

Choose any three-digit number (where the units and hundreds digit are not the same).

Subtract the number with the digits reversed.

To this difference, add the numbers with the digits reversed.

Your result should be 1089. Why?



Precautions and Possible Pitfalls

The primary precaution when doing a motivational activity is to make sure that it is appropriate for the intended students in both interest and level. In addition, ensure that it leads to the topic for which you are motivating your students rather than distracting them from it. If successful, try to modify other lessons to maximize student interest so that the usual classwork does not become boring compared to this highly interesting activity.

Sources

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Strategy 24: Use school fund-raising projects, such as students’ selling candy or organizing a walkathon, as the basis of mathematics lessons.



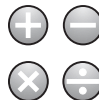
What the Research Says

Fund-raising activities, such as students selling candy or organizing a walkathon to raise money for a trip or special equipment, can be used as the basis of meaningful mathematics learning. One study examined how children develop and use mathematical knowledge through their experiences selling candy. This research showed that the hands-on experience of acting as a salesperson helped students to learn and understand important mathematical knowledge that they were later able to apply to working on school problems.



Teaching to the National Council of Teachers of Mathematics Standards

The National Council of Teachers of Mathematics *Handbook of Research on Mathematics Teaching and Learning* devotes a chapter to “Ethnomathematics and Everyday Cognition.” In this treatise, ethnomathematics is defined to include mathematics that is involved in “everyday activities such as building houses, exchanging money, weighing products, and calculating proportions for a recipe.”⁴ Although the focus of the research showed how “nonmathematical” activities in the home required the use of mathematics and problem-solving skills, it can also be said that there are situations



in school that occur outside of the mathematics classroom that support the use of mathematics. The following application suggests that students should participate in school activities that require the use of mathematics and support the classroom instruction, albeit in a nonconventional manner. These real-life activities may be more meaningful to students than the typical textbook exercises that are assigned for homework.



Aligning to the Common Core State Standards

In addition to using “real-life” data, you can use actual experiences in the school community to create mathematical problems and lessons. When students are modeling their own world with mathematics, they are invested in a deeper way. Students can use and apply what they’ve learned to what is going on in their school. They can make sense of the world around them when you bring school-related activities to the classroom.



Classroom Applications

If there is a school store in your school, then it would be a good idea to make contact with the store manager and offer your class’s services to do the accounting for the operation. This would then require getting all the purchasing and sales information about the store and then having the class decide how to manage the information. If there is no school store, then you might start one or get the principal’s permission to undertake a project of fund-raising. Funds so generated may be to improve the school or to purchase important equipment for the school, such as band uniforms or computers. Students could also combine mathematics with social studies content by raising funds to be contributed to a local charity for homeless people. Once the project has been approved and the plan set, let students calculate the cost of doing business, the price of the items to be sold, and the anticipated profit based on specific sales results.

More sophisticated mathematics (or higher-order thinking skills) can be seen in students’ estimating how much money they would generate if they offered different discount rates on the items they were selling. Students could compare the relative benefits of conducting different types of fund-raising events, such as bake sales versus T-shirts. In addition, they could make projections for next year’s fund-raising target based on current data. A

student-sponsored walkathon is also another activity that can provide rich mathematical content. Students can engage in calculations with measurement, geometry, unit conversion in addition to the many other team-building skills that accompany any large-scale student-led initiative. Finally, they could compare the success of school fund-raising strategies with those of non-school fund-raisers, such as local churches sponsoring benefit dinners or raffle ticket sales.

Precautions and Possible Pitfalls

Care must be taken to ensure that the activities undertaken support the mathematics program and that the “business” doesn’t take on a life of its own, where the main (original) purpose to motivate and excite students about mathematics gets lost.



Source

Saxe, G. B. (1988). Selling candy and math learning. *Educational Researcher*, 17(6), 14–21.

Strategy 25: When doing inquiry lessons, give students clearly written materials to guide the inquiry process.

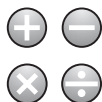


What the Research Says

Research on the inquiry-based computer program Geometric Supposers was conducted in 23 high school geometry classes. This program, Geometric Supposers, uses teacher-posed inquiry problems. The researchers specifically designed the materials so they would clearly communicate to the students what a particular problem is and what appropriate inquiry activities are expected. Evidence was collected from six sources: classroom observations, student interviews, students’ work on the Supposers program, teacher interviews, teacher reflections, and minutes from monthly teachers’ meetings. The results showed that (1) clearly written materials mean students will understand what work needs to be done, (2) they help students organize their work, and (3) using charts and tables that tell students which measurements to make and giving step-by-step instructions were ineffective because they



limited their inquiry. These findings led the researchers to conclude that how inquiry materials are written affects students' success in inquiry.



Teaching to the National Council of Teachers of Mathematics Standards

The National Council of Teachers of Mathematics *Handbook on Mathematics Teaching and Learning* devotes much attention to learning geometry at various grade levels using inquiry-based lessons. The use of dynamic software and computer environments shows results that are “intriguingly consistent. Difficulties and misconceptions that are easily masked by traditional approaches emerge in computer environments.”⁵ Thus, as the following application suggests, students must be given a precise roadmap upon which the inquiry can be based. Appropriately designed inquiry lessons can provide a high level of conceptual understanding of geometric ideas, and the students will achieve a sense of autonomy in their mathematical thinking.



Aligning to the Common Core State Standards

Standard for Mathematical Practice #4 says that students must “Model with Mathematics.” They encourage teachers to expose students to how mathematics relates to real-life situations. “Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.”⁶ One of the most oft-cited criticisms of secondary mathematics is the lack of authenticity. Providing students with the opportunity to inquire into their world is paramount to understanding and providing clear guidelines and direction will allow students to be comfortable with their task. Inquiry work is more self-directed, so the materials and instructions provided will put them at ease through the process. This isn’t always easy, so use your network of peers to test out the instructions so you can troubleshoot any questions that arise before distributing an inquiry-based lesson or unit to your class.



Classroom Applications

Teachers should prepare clearly written materials for students, making sure to leave an opportunity for students to

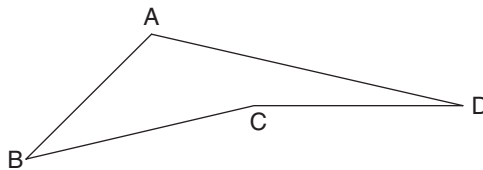
conduct an inquiry. Three strategies for writing clear materials are as follows:

1. State the goal of the problem at the top of the page.
2. Provide explicit instructions on the processes to use when solving the problem so students know how to engage in inquiry, but don't provide so much structure that it cuts off student inquiry and it turns into students' collecting data or following directions without understanding what they are doing.
3. After students understand how diagrams can be used as models, use diagrams to simplify written instructions.

Sample Goal Statement

Task: To develop a procedure that enables you to reproduce or reconstruct Figure 4.1.

Figure 4.1



Sample Process Instructions

Procedure:

Make a drawing similar to this figure.

Collect data.

Describe below the procedures for reproducing this figure.

State your conjectures.

Precautions and Possible Pitfalls

Written material that is not clear prevents students from making successful inquiries. When creating inquiry problems for students, teachers should consider three issues before writing the problem statement and three issues while writing it. The three



issues to consider before writing the problem statement are the kind of problem, its scope, and the students' background or ability. Three issues to consider while writing the problem statement are stating the goal of the problem, describing any constructions in the problem, and giving instructions for inquiry processes students are expected to use.

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Strategy 26: Use graphic representations or illustrations to enhance students' memory while they are listening to you. Abstract representations such as flowcharts are more effective than colorful pictures.



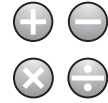
What the Research Says

There are several types of supplementary materials that can help students remember what they learn while listening to a teacher. Some types of materials are better than others. One study compared the effect on students' memory of graphic representations (flowcharts with keywords) versus colorful pictures. The study was conducted with 23 girls and 33 boys, aged 11 to 13 years. The entire sample was separated into four groups. In each group students listened to a tape recording of textual material. The four types of material were as follows:

1. No graphic representation and no picture
2. Graphic representation (flowchart with keywords from the text)
3. Colorful picture
4. Graphic representation and picture

After hearing the tape, students completed a questionnaire to check the effects of the different types on students' memory. The results showed that students remembered more when they were given the graphic representations.

Teaching to the National Council of Teachers of Mathematics Standards



A *Research Companion* to the National Council of Teachers of Mathematics Standards devotes much attention to graphical representation in an expository piece titled "Learning to Graph and Graphing to Learn." The focus of this piece centers upon using the graphical representations as a means to engage students in the "broader communication process." This process should support and complement the auditory process and must be carefully chosen not to interfere with it. The article concludes with the statement:

Graphing is no longer a topic consisting of a few skills and procedures to be taught once and for all. As a means of communication and of generating understanding, graphing must repeatedly be encountered by students as they move across the grades from one area of school mathematics to another.⁷

Aligning to the Common Core State Standards



Beginning in the second grade, the CCSSM encourage the use of visuals and manipulatives to support implementation of data and quantity. Students are challenged to become fluent in using representations from an early age, so we should continue this strategy at the secondary level. The CCSSM envision students who are "are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas."

Classroom Applications



Pictures can inhibit memory of knowledge given auditorily. The complexity of a picture takes too much visual attention and stresses the capacity of intellectual processing. When students read a text, then pictures can support the verbal information, because during reading a person can focus her or his attention either on text or on picture and can move ahead at her or his own pace. However, pictures can interfere with listening. They can interrupt the logical

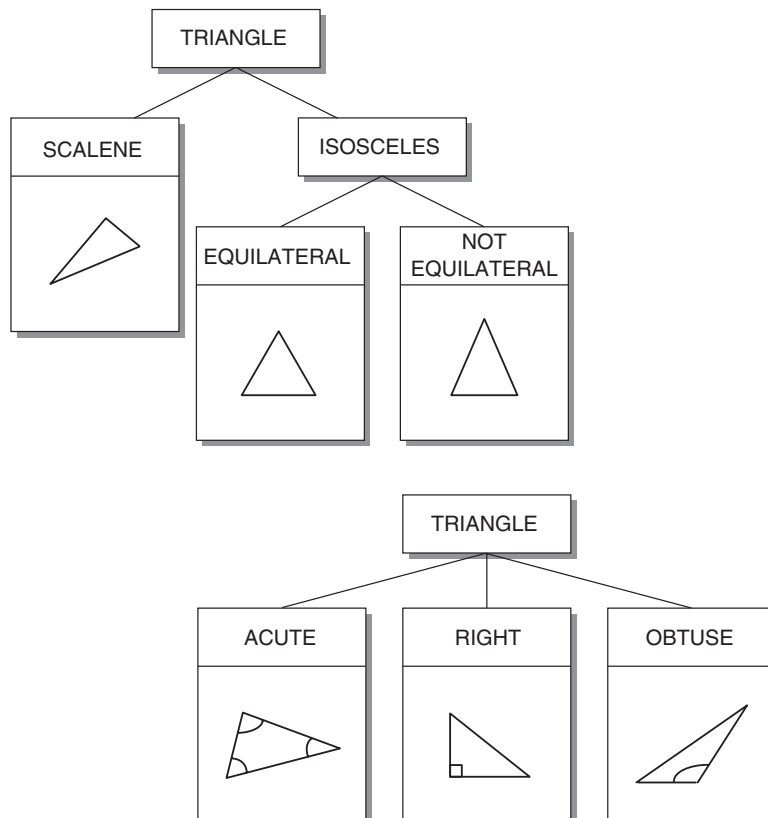
flow of the words. Pictures and words go together only when the auditory information explains the picture.

There are many types of graphic representations mathematics teachers can use. They include the following:

- Flowcharts
- Rough structural sketches
- Continuums
- Matrices
- Venn diagrams
- Tree diagrams
- Concept maps
- Problem-solution charts

Graphic representations are characterized by being quickly understood, providing a structure for integrating new information, and being schematized sketches instead of colorful pictures. Figure 4.2 is an example of an effective graphic representation.

Figure 4.2

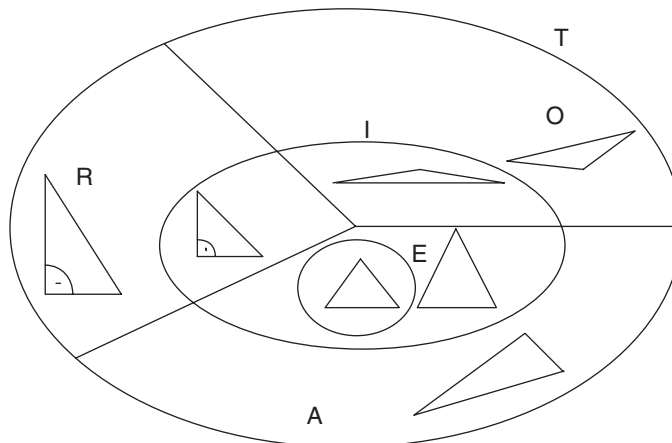




Precautions and Possible Pitfalls

The illustration in Figure 4.3 demonstrates that not every graphic representation aids learning and memory. The representation in Figure 4.3 draws too much attention away from what students will be listening to because it is too complex. It combines several characteristic classifications into one representation. That is why it is recommended to construct a graphic representation as simple as possible if students are to see it while they are listening. A complex representation such as the one in Figure 4.3 may be useful for students when they are not listening to a lesson.

Figure 4.3



- T: set of all triangles
- A: set of all acute triangles
- O: set of all obtuse triangles
- R: set of all right triangles
- I: set of all isosceles triangles
- E: set of all equilateral triangles

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Strategy 27: Playing makes understanding mathematics easier and more fun.



What the Research Says

Research repeatedly has demonstrated the learning effects and motivating outcomes of lessons involving play. One study with third- to sixth-grade students examined the impact of play on achievement in mathematics. The results showed that in addition to students learning more, their active participation and motivation increased during the following class period.



Teaching to the National Council of Teachers of Mathematics Standards

The National Council of Teachers of Mathematics Problem Solving Standard calls for students to “apply and adapt a variety of appropriate strategies to solve problems.”⁸ Games are a very effective way to get students to engage in problem solving. Most games are based primarily in mathematics. Students who recognize the underlying mathematics and begin to strategize using mathematical representations are more likely to become proficient in both the game and the underlying mathematics. The game becomes a mental manipulative of the mathematics while the mathematical problem solving supports winning strategies of the game. Linking games to mathematics is simply the merging of informal knowledge with the mathematics of the day’s lesson. It is a time-tested way to excite students about learning mathematics while highlighting the relevancy of the mathematics that they are learning.



Aligning to the Common Core State Standards

Students are attracted to games and manipulatives, so using these methods to model mathematics is a natural application. Although you may use games to discuss probabilities and riddles to understand numeracy, you can also use sports and trivia to help

students apply the content. For example, Standard 7.SP.4-5 expects students should be developing probability models in seventh grade, and teachers in each grade level can build the level of difficulty as each grade level progresses.

Classroom Applications

The activity described next, which is like a game show, was used in the experiment described earlier. You may adapt it for your students and mathematical content.



1. Preparation for the Quiz

- Project on the digital whiteboard or create a transparency with a basic diagram with 36 squares (see Figure 4.4).
- Six questions for luck and six questions for risk are distributed at random on a hidden copy of the basic diagram (see Figure 4.5). Make sure you have enough of the luck and risk questions. Students like them, and they motivate students to play.

Figure 4.4

The Big Prize

	1	2	3	4	5	6
A						
B						
C						
D						
E						
F						

Figure 4.5

The Big Prize

	1	2	3	4	5	6
A		<i>Luck</i>			<i>Risk</i>	
B				<i>Risk</i>		<i>Luck</i>
C			<i>Risk</i>	<i>Luck</i>		
D	<i>Luck</i>	<i>Luck</i>				
E			<i>Risk</i>		<i>Risk</i>	
F	<i>Luck</i>					<i>Risk</i>

2. Preliminary Round

- Every student has to answer three questions.
- Each student is allowed to personally select the mathematical topic (e.g., oral multiplication).
- Every correct answer earns two points for the student.
- Six points is the maximum per student.

3. Main Round

- Every student starts the main round with the number of points he or she has accumulated in the preliminary round.
- As soon as the moderator asks a question, students can raise their hands to answer.
- The student who raises his or her hand first is allowed to answer the question.
- The student who earned the previous points has to select the square for the next student.
- Each Luck square gives five points for the correct answer.
- Each Risk square allows the student to as much as double the number of points he or she has accumulated. The student

decides how many points to risk. If the student correctly answers the question, the points that are risked will be added. If the student gives an incorrect answer, the number of risked points will be subtracted from his or her score.

- The remaining squares provide one point added for each correct answer and one point subtracted for each wrong answer.

4. Course of the Quiz (This can be given out to students)

- Class and teacher elect people for the following roles:
 - moderator (this could be the teacher)
 - assistant (covers the answered squares)
 - three counters (scorekeepers)
 - referee (decides on the order of students, who raised their hands first)
 - six game players
- Arrange the chairs in a horseshoe. Players can sit at the front side and the “audience” at the other sides.
- The counters announce the score at the end of the preliminary round.
- The player who has the highest score starts the main round. In case of a tied score between several students, the winner is decided by the results of a play-off question.
- The transparency with the basic diagram (Figure 4.4) lies on an overhead projector. The moderator keeps the transparency that appears in Figure 4.5.
- The player who starts the main round chooses one square (e.g., square D4).
- The assistant covers this square with a stone or the like.
- The moderator asks the question.
- According to the order of raised hands, students give their answers until the correct answer is given. The referee observes the correct order and tries to see if anyone is cheating.
- If a square with Luck or Risk is selected, only the student who has chosen it is allowed to answer the question. Only if he or she gives a wrong answer does the question become open to the other players.
- The quiz is finished when all squares are gone.
- The student with the highest score wins.

Besides written tests, a teacher should look for fun and efficient ways to check knowledge, abilities, and skills.



Precautions and Possible Pitfalls

Playing games during lessons requires constant and strict organization, otherwise you will produce more problems than motivation. Give very precise directions for playing the game and for how you expect students to behave. In case students start to misbehave, stop the game and try it again during one of the next lessons.

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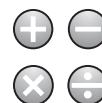
Strategy 28: Assign homework and other projects requiring students to write about connections between mathematics and other subjects.



What the Research Says

The Office of Educational Research and Improvement at the U.S. Department of Education funded the development of a digest to assist teachers in the difficult task of making connections between mathematics and other subjects. Research indicates that making such connections is especially difficult at the high school level, where students have different teachers for different subjects and there is a strong emphasis on distinct content areas. The National Council of Teachers of Mathematics recommends making such connections, but teachers often do not have the knowledge or resources needed to implement this recommendation. Research indicates that when students connect their mathematical knowledge and skills with other subjects, mathematics is seen as more interesting and more useful than when students see mathematics as a separate subject.

Teaching to the National Council of Teachers of Mathematics Standards



The National Council of Teachers of Mathematics Standards for Curriculum and Evaluation state that the “mathematics curriculum should include investigation of the connections and interplay among various mathematical topics and their applications so that students can use and value the connections between mathematics and other disciplines.”⁹ As the application below suggests, connections can easily be made to real-life applications. Sports and business sections of the newspaper provide real-life connections to mathematics. Asking students to discover other connections should be encouraged; students should find less obvious connections to mathematics, like the interpretation and manipulation of statistics to support differing positions and ideas.

Aligning to the Common Core State Standards



In one lesson, we asked students to draw a map of their neighborhood for homework and identify angle pairs using the intersections of streets. This homework assignment allowed them to take the idea and visualize how their world is connected to the mathematics we were learning. In a lesson on three-dimensional figures, students had to go home and take inventory of three items in their home that matched the vocabulary we were studying. Instead of completing a “worksheet” or copying answers from textbook questions, students were able to share their stories of exploration and discovery. The CCSSM encourage you to create students who “can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.”

Classroom Applications



Teachers often feel that they do not have enough time in class to help students connect what they are learning about mathematics with other subject areas, especially when they want students to read and write about these connections. Homework and other out-of-class assignments, such as reading newspapers and magazines, can be excellent ways of addressing this limitation. A resourceful teacher will be able to show students how almost every page of a newspaper contains applications of mathematics. Naturally, the sports pages and the business section

are obvious illustrations of mathematics at work in subjects of gymnastics and history or social studies. These connections, although well known to most students, are not always obvious. For example, a “batting average” is actually not an average in the way mathematicians understand the term; rather, it is a percentage or decimal. The most exciting discoveries of mathematics in use in the newspaper are those where the mathematics is embedded in the article, not necessarily those illustrations that give some quantifiers and the like. Asking students to regularly search for good newspaper applications of mathematics and their review in class are excellent ways of connecting mathematics across the curriculum. Students can also research and write about the relationships between mathematics and related topics in other subjects such as elections, taxes, and the stock market in history or social studies; mixtures of solutions and soil composition in science; fractals in art; as well as scales, chord structures, and chord progressions in music.



Precautions and Possible Pitfalls

One should be careful of using, as connectors to other fields, the trivial, or simple, illustrations of numbers being used to quantify a situation. Students may like to use these as their examples, but since they offer relatively little to connect mathematics to other fields, it would be wise to caution the class not to use these as their examples.

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Strategy 29: Encourage students to make mental pictures while applying rules to solve problems.



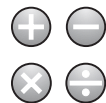
What the Research Says



Creating and using mental images about rules can help students solve problems. A study was conducted with 52 ninth-grade students who were assigned to one of two groups. Both groups were given information on Boyle's Law, Charles' Law, and Gay-Lussac's law. One group of students was encouraged to create mental images of a typical gas as it responded to different amounts of pressure, temperature, or volume. This group was also instructed to draw an image of this in their notebooks. Students in the other group were not instructed to make mental pictures or draw images in their notebooks. These students were told to write the rule and repeat it out loud while learning. After three days of instruction on the gas laws, students were given an exam and completed a questionnaire designed to assess their use of imagery while they were taking the exam. The exam consisted of two parts. The first part was made up of six multiple-choice items designed to measure students' memory. The second part consisted of six essay questions in which students had to use the rules to solve problems. Students had to solve these problems correctly and give acceptable explanations for their answers.

The results showed that the imagery group did better than the nonimagery group in solving the problems, but the nonimagery group did better than the imagery group on the memory part of the exam. However, many other studies have demonstrated mental imagery can have a very beneficial impact on memory.

Teaching to the National Council of Teachers of Mathematics Standards



The National Council of Teachers of Mathematics Professional Teaching Standards of 1991 state that "the teacher of mathematics should engage in ongoing analysis of teaching and learning by examining effects of the tasks, discourse, and learning environment on students' mathematical knowledge, skills, and dispositions."¹⁰ This suggests that teachers must be very sensitive to the various types of learning tasks in which their students are engaged. Because the use of imagery was profoundly effective for

students applying rules to conceptual ideas, it should be employed as a teaching Strategy in such cases. Similarly, repetitive activities effectively supported the type of learning that required memorization. Although repetitive classroom activities that support memorization are typically discouraged, there are times when they are both necessary and advisable. Teachers should use multiple strategies to support the different learning tasks of their students.

The National Council of Teachers of Mathematics Problem Solving Standard calls for students to “apply and adapt a variety of appropriate strategies to solve problems.”¹¹ Thus, procedural knowledge should be strengthened with conceptual understanding that is enhanced by visualization and imagery. The following applications are wonderful examples of employing simple visualization techniques to simplify complex problems. Encouraging students to use a variety of strategies to solve problems is consistent with the standards and is a good practice in their daily lives.



Aligning to the Common Core State Standards

We will make every effort as educators to provide authentic and engaging modeling problems for our students, but sometimes we can simply ask students to apply their own interpretations to create models for themselves. We can teach students to seek out mathematics in their own lives and allow them to enrich the curriculum through their own experiences and thoughts. Allowing quiet time and space for students to use their mind to visualize their problem solving will create brain synapses that they might not if we are consistently providing data for them to interpret and apply.



Classroom Applications

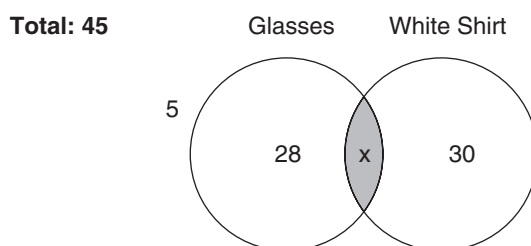
The preceding research has clear implications for mathematics instruction. For learning algorithms, it makes sense to have students remember the rule being used. Perhaps later, a more in-depth study of the algorithm can be helpful. For the solution of problems that, by their very nature, do not call for a diagram, it is sometimes quite helpful to visualize the situation being described. Visualization can be in the form of sketches, diagrams,

or mental pictures. A “mental picture” is a thought diagram. Consider the following problem:

In a room of 45 children, 28 wear eyeglasses, 30 are wearing a white shirt, and 5 are not wearing a white shirt and do not wear eyeglasses. How many are wearing a white shirt and have eyeglasses on?

To do this problem most expeditiously, it would be wise to mentally picture the students and draw a Venn diagram of the situation. Figure 4.6 would provide students with a visual representation that would guide them in solving this problem.

Figure 4.6



The intersection x can be found by the equation: $(28 - x) + (30 - x) + x + 5 = 45$, arrived at by adding the contents of each of the regions, which total 45. Therefore, $x = 18$. Without drawing the diagram the problem would have been considerably more difficult. Nothing in the statement of the problem told the students to make the drawing. This is where the teacher’s role is important. Teachers should directly instruct students in making drawings and/or mental pictures of problems they are solving. Students need to be encouraged and reminded to make and use mental images and diagrams.

In addition to drawing, students can form mental pictures of problems just by using their visualization skills. For example, when reading a problem like “Max is taller than Sam, and Jack is taller than Max. Who is the tallest?” a student can make mental pictures of Max, Sam, and Jack to compare their relative heights.



Precautions and Possible Pitfalls

Drawing pictures to represent abstract concepts or topics is useful but is by no means the solution to all problems. Teachers should be careful to make this point clear. Yet, they should always encourage students to try to represent the problem situation graphically to gain more insight. Many students view mental images as “crutches” or forms of cheating and feel guilty about using them. Let students know it’s smart to use such strategies.

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