Operations and Algebraic Thinking
4.OA.A*

Cluster A

Use the four operations with whole numbers to solve problems.

STANDARD 1 **4.OA.A.1:** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

STANDARD 2 **4.OA.A.2:** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]

[1] See Table 2 in the Resources, page 256.

STANDARD 3 **4.OA.A.3:** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

*Major cluster

Operations and Algebraic Thinking 4.OA.A

Cluster A: Use the four operations with whole numbers to solve problems.
Grade 4 Overview

Fourth graders have worked with equal group and array/area problem situations for multiplication and division in Grade 3. Multiplication and division comparison situations are introduced in Grade 4. Students continue to work with one- and two-step problems that use all four operations, including problems in which remainders must be interpreted in terms of the question being asked in the problem.

Standards for Mathematical Practice
SFMP 1. Make sense of problems and persevere in solving them.
SFMP 2. Use quantitative reasoning.
SFMP 3. Construct viable arguments and critique the reasoning of others.
SFMP 4. Model with mathematics.
SFMP 5. Use appropriate tools strategically.
SFMP 6. Attend to precision.
SFMP 7. Look for and make use of structure.
SFMP 8. Look for and express regularity in repeated reasoning.

Students at this level use quantitative reasoning to solve single and multi-step problems that include all four operations using models, pictures, words, and numbers. In addition to equal group and area situations, they begin to solve multiplication and division comparison problems. They think about solutions in terms of reasonableness, asking themselves "Does this make sense?" Estimation strategies not only help to extend conceptual understanding but also students' thinking about the numbers in a problem to determine whether a solution is reasonable. Students explain their thinking using concrete materials, pictures, words, and numbers. They listen to the reasoning of others and look for similarities and differences in various strategies used to solve a problem. Using appropriate mathematical vocabulary and accurate units of measure are areas of focus as students begin to solve more sophisticated problems.

Students use various representations and models to help solve problems. They continue to develop problem solving strategies, including make a model, draw a picture, make an organized list, find a pattern, solve a simpler problem, and guess and check. Students select appropriate tools, including concrete materials, graph paper, and pictures to help solve problems. They also ask themselves whether a task can most efficiently completed by mental computation, estimation, or paper and pencil. For more complex situations, they might use a calculator.

Students look for and extend mathematical patterns in a variety of situations and connect those patterns to the properties. These patterns help students understand the structure of the four operations and should also be connected to the work in the Number and Operations in Base Ten (NBT) domain.

Related Content Standards

2.OA.A.1 2.MD.B.5 2.MD.C.8 3.MD.A.1 3.MD.A.2 3.MD.B.3 3.OA.D.8 3.MC.C.7 3.G.A.1

4.MD.A.2 4.MD.B.4 4.MD.C.7 5.MD.C.5 5.G.A.2

Notes

STANDARD 1 (4.OA.A.1)

Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

In the earlier grades students worked with additive comparisons. Megan has 25¢ and Liz has 5¢. How much more does Megan have than Liz? (2.OA.1). (What amount would be *added* to Liz's money to get Megan's amount?)

Fourth graders learn to compare these quantities multiplicatively. Megan has 5 times as much money as Liz. (What factor would *multiply* Liz's money to get Megan's amount?) A multiplicative comparison is a situation in which one quantity is described as a multiple of the other.

Another way to identify multiplicative comparisons is recognizing that in these situations there are two different sets being compared. The first set contains a certain number of items. The second set contains multiple copies of the first set. The language of multiplicative situations can be difficult for students. Students should become familiar with these and similar ways to describe multiplicative comparison situations.

Darlene has seven marbles. Danny has 3 times as many.

Danny has 3 times as many marbles as Darlene.

The number of marbles Danny has divided by 3 is the number of marbles Darlene has.

Danny has 3 times fewer marbles than Darlene.

This Standard should be taught in concert with 4.OA.A.2 so that students are consistently working among problem situations, models, and equations as they work with comparisons.

What the TEACHER does:

- Provide many opportunities for students to identify and model multiplicative comparison situations (Table 2, page 256).
- Reinforce appropriate vocabulary to describe comparison situations and appropriate examples for writing equations from these situations.

What the STUDENTS do:

- Read and interpret multiplicative comparison situations identifying which quantity is being multiplied and which factor is telling how many times.
- Write and identify equations and statements for multiplicative comparisons.

  5×3 Cathy has \$5. Mary has three times as much. How much money does Mary have?

- Recognize different language that describes multiplicative comparisons. (See Table 2, page 256.)

Addressing Student Misconceptions and Common Errors

Students may struggle with applying their knowledge of multiplication and division facts to multiplicative situations since all of their previous experience was with equal groups and array models. They need many experiences connecting facts to the language of multiplicative comparisons. Using concrete models will support students in making this connection.

Notes

STANDARD 2 (4.OA.A.2)

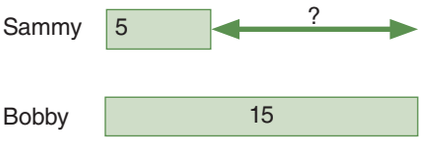
Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]

[1] See Table 2 in the Resources, page 256.

In previous grades students worked with tape diagrams and other models to show additive comparison situations (see Table 1, page 254). Similar models with different representations will help with understanding multiplicative comparison situations. Early problems should involve one step and then build to multiple-step problems that involve up to three steps.

Additive Comparison (5 + m = 15)

It takes Sammy 5 minutes to wash the dishes. It takes his brother Bobby 15 minutes to wash the dishes. How much longer does it take Bobby to wash the dishes?



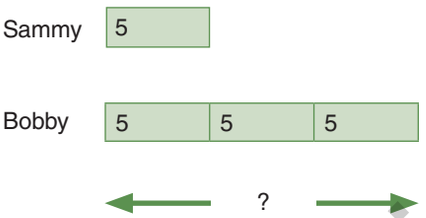
In this situation, we are finding the difference between Sammy's time and Bobby's time.

Multiplicative Comparisons

Product unknown: (3 × 5 = *t*)

It takes Sammy 5 minutes to wash the dishes. It takes his little brother Bobby 3 times as long. How long does it take Bobby to wash the dishes?

Think: 5 minutes 3 times would be?

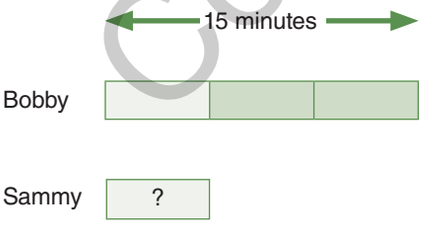


In this situation, Bobby's time is a multiple of Sammy's time.

Factor unknown (size of each group unknown) 3 × m = 15

It takes Bobby 15 minutes to wash the dishes. That is three times as long as it takes his brother Sammy. How long does it take Sammy to wash the dishes?

Think: Three groups make 15, how big is each group?



Factor unknown (number of groups unknown) $g \times 5 = 15$

It takes Sammy 5 minutes to wash the dishes and his little brother Bobby 15 minutes to wash the dishes. How many times as long does it take Sammy to wash the dishes?

Think: How many 5s to make 15?

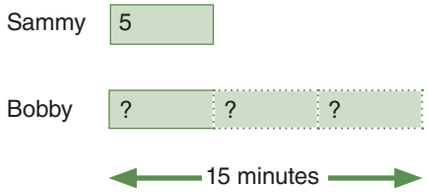


Table 2, page 256 in the Resources, provides various examples and models for comparison situations.

What the TEACHER does:

- Provide a variety of multiplicative comparisons (see Table 2, page 256) for students to model, describe, and solve.
- Facilitate explicit discussions in which students describe the information in the problem and use that information to represent the situation using models.
- Help students to make explicit connections between models (such as bar models), and written equations using both multiplication and division.
- Provide mixed additive and multiplicative comparison situations to help students distinguish between the two types of comparisons.
    - Additive comparisons focus on the difference between two quantities.
        - How many more?
        - How many less?
    - Multiplicative comparisons focus on comparing two quantities when one is a specified number of times greater or less than the given quantity.

What the STUDENTS do:

- Solve problems involving multiplicative comparisons using concrete materials, pictures, words, and numbers.
- Identify the information in the problem and how it relates to models.
- Write equations to represent the mathematics of the situation.

Addressing Student Misconceptions and Common Errors

Students may confuse additive and multiplicative situations. They need a variety of problems to model and discuss. Identifying what they know from the information in the problem and focusing on the question will help them to make sense of the problem. They should also consistently ask themselves if their answer makes sense.

Notes

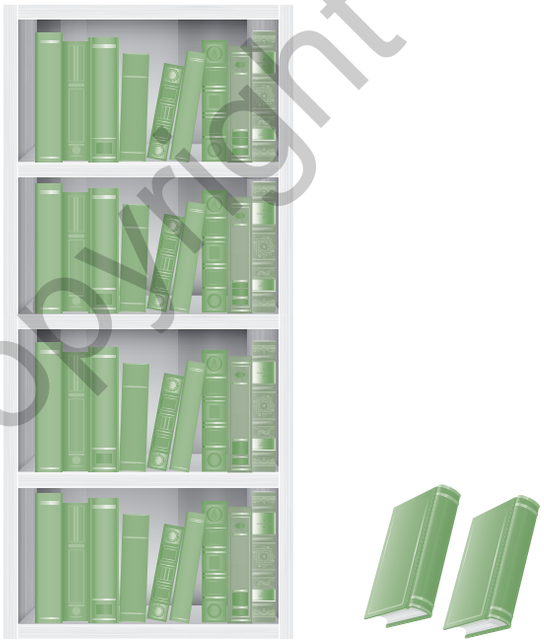
STANDARD 3 (4.OA.A.3)

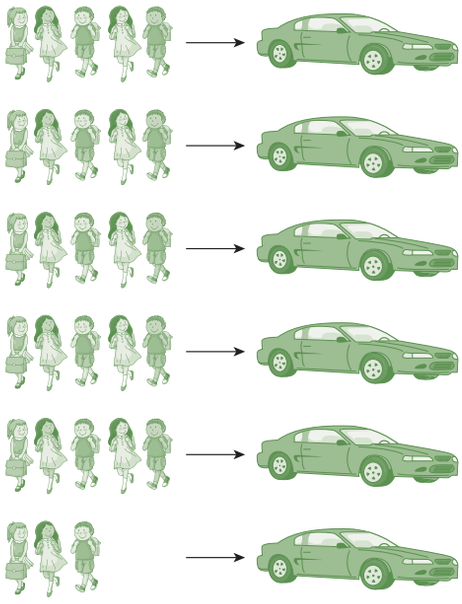
Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Students continue to work with problem situations as they build fluency with all four operations. Some students may still need to use concrete and pictorial models and connect these models to numerical equations representing the unknown with a letter before solving. Others may be able to determine the equation by recognizing the information in the problem and the question that is being asked.

Students should consistently use formal and informal estimation strategies to determine whether an answer is reasonable and fits the constraints of the given situation. They should have many opportunities to explain their strategies to predict a reasonable solution or to justify why they think their answer is reasonable. Remember, good discussions take time, so it is important to have rich conversations around a small number of good problems rather than superficial discussions about a large number of problems.

This will be students' first experience with interpreting remainders. Using models, identifying the information in the problem, focusing on the question, and discussing the reasonableness of results will help students determine what to do with a whole number remainder. (They will use remainders as fractions and decimals in later work.) It is important that students have ongoing experiences with a variety of problems in which they need to determine what to do with the remainder. Encouraging students to explain their reasoning in solving such problems will also help them to think about what to do with the remainder.

Problem	Model	Equation	What to do with the remainder
Eric bought a package of 18 batteries. Each toy robot uses 4 batteries. How many toy robots can be filled with batteries?		$18 \div 4 = 4$ remainder 2 That means that 4 robots can be filled with batteries and I will have 2 batteries left over.	The solution is the quotient. Drop the remainder.
Alyssa has a new bookcase with 4 shelves. Each shelf holds 9 books. If Alyssa had 38 books how many books will not fit in the bookcase?		$38 \div 4 = 9$ remainder 2 Two books will not fit on the shelves.	The remainder is the solution.

Problem	Model	Equation	What to do with the remainder
Twenty-eight students are going on the class picnic. Five students can ride in each car. How many cars will be needed for the trip?		$28 \div 5 = 5$ remainder 3 That means 5 cars can hold 25 students and another car will be needed for the extra 3 students. Six cars will be needed for the trip.	Add 1 to the quotient for the solution.

What the TEACHER does:

- Provide ongoing experiences with problems, including two- and three-step problems with all four operations.
- Facilitate small group and classroom discussions in which students show and explain their strategies and solution processes using materials or pictures, words, and numbers.
- Build on previous division problem experiences by including division problem situations that include remainders.
- Lead class discussions on what to do with the remainder by focusing on the problem question, the meaning of numbers in the problems and using models.

What the STUDENTS do:

- Solve multi-step problems with all four operations using models or pictures and numbers.
- Explain their problem solving processes and compare various ways of solving problems.
- In division situations with remainders, focus on the question asked to determine what to do with the remainder.
- Ask themselves if their solution makes sense.

Addressing Student Misconceptions and Common Errors

Students who struggle in determining what operation to use to solve a problem need additional experience understanding the operations in a variety of situations (see Tables 1 and 2, pages 254 and 256). They should have explicit practice with various problem solving strategies, including:

- Restating the problem in their own words.
- Identifying given, needed, and wanted information.
- Making a model or drawing a picture.
- Making a list.
- Acting it out.
- Finding a pattern.
- Writing an equation.
- Revisiting the question and asking themselves if the solution makes sense.

Students who do not have conceptual understanding or have focused work on division procedures tend to write answers to problems using the "r" notation. For example, 16 students are going canoeing. If each canoe holds 3 canoes, how many canoes will they need? The answer 5 r 1 makes no sense in this situation. Students need to focus on the question and reasonableness of solutions using strategies including models, pictures, and acting it out.

Operations and Algebraic Thinking
4.OA.B*

Cluster B

Gain familiarity with factors and multiples.

STANDARD 4

4.OA.B.4: Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

*Supporting cluster

Operations and Algebraic Thinking 4.OA.B

Cluster B: Gain familiarity with factors and multiples.
Grade 4 Overview

Students extend their understanding of multiplication and division to thinking about these operations in terms of composing and decomposing numbers into factors. For example, 12 can be decomposed into factors of 1, 2, 3, 4, 6, and 12 by knowing the multiplication facts that result in a product of 12. Making arrays will help students to build understanding of factors, reinforcing fluency with basic facts and extending to factor pairs beyond the basic facts. This Standard extends this understanding through recognizing prime numbers (numbers with exactly two factors) and composite numbers (number with more than two factors).

Standards for Mathematical Practice
SFMP 2. Use quantitative reasoning.
SFMP 3. Construct viable arguments and critique the reasoning of others.
SFMP 4. Model with mathematics.
SFMP 5. Use appropriate tools strategically.
SFMP 6. Attend to precision.
SFMP 7. Look for and make use of structure.
SFMP 8. Look for and express regularity in repeated reasoning.

Students extend their work with multiplication and division facts to focusing on finding factors and multiples of numbers less than 100. Although their facility with facts will help, using models and reasoning with the distributive property will help them find the factors that are not basic facts. For example, the factor pairs for 36 are 1×36, 2×18, 3×12, 4×9, and 6×6. Of these factor pairs, only 4×9 and 6×6 are considered basic facts.

Once students understand factors and multiples, they build on this understanding by defining and identifying prime and composite numbers. These concepts are important to future work with fractions. Students construct arguments based on the patterns they have found, including, for example, why 45 is a composite number and 47 is prime. They do this using models, words, and numbers.

Students extend their knowledge of basic facts by constructing arrays for numbers beyond the basic facts. For example, they can use a model of an 11×3 array to show that 3 and 11 are factors of 33 using graph paper, square tiles, and other appropriate tools. As they complete these activities, the vocabulary of multiplication is extended to include *factor*, *multiple*, *prime number*, and *composite number*. Students should be able to clearly define these words and use them in a variety of contexts.

Students recognize patterns as they explore numbers. Some numbers have exactly two factors and others have more than two factors. They will apply these structures and they begin to work with fractions. They use these patterns to make and justify generalizations such as "all even numbers other than 2 are composite because they will have more than two factors." or "All numbers other than 5 that end in 0 or 5 are composite because they will have 5 as a factor."

Related Content Standards

3.OA.B.6 3.NBT.A.3 4.NF.B.4.a 4.NF.B.4.b

STANDARD 4 (4.OA.B.4)

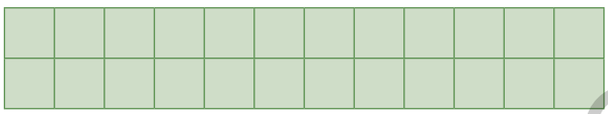
Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

This Standard builds on and extends students' knowledge of multiplication and division facts. Factor pairs include two numbers that when multiplied result in a particular product. The factor pairs of 28 include 1 × 28, 2 × 14, and 4 × 7, so the factors of 28 are 1, 2, 4, 7, and 28. Students can use square tiles to make arrays to find all of the factor pairs of a given number. They then explore patterns to build a conceptual understanding of prime numbers (numbers with exactly two factors) and composite numbers (numbers with more than two factors).

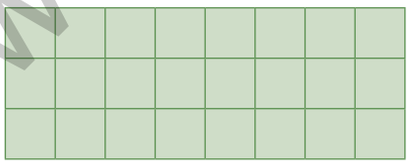
Multiples are the result of multiplying two whole numbers. Multiples can be related to factors, as shown below. Skip counting by a given number also results in the multiples of that number.

Using arrays to find the factors of 24.

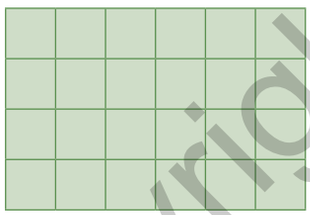
1 × 24



2 × 12



3 × 8



4 × 6

The *factor pairs* for 24 are 1 × 24, 2 × 12, 3 × 8, and 4 × 6.

The *factors* of 24 are 1, 2, 3, 4, 6, 12, and 24.

12 is a *multiple* of 1, 2, 3, 4, 6, and 12.

This Standard connects to 4.OA.C.5 in that it provides a variety of contexts for discussing number patterns that arise when students find factors and multiples. It is important to providing opportunities for students to use concrete materials, hundreds charts, and fact tables to discover patterns, followed by oral and written explanations describing those patterns, helps students develop a deeper understanding of factors, multiples, primes, and composites and to begin to make generalizations about the patterns they have found.

Such patterns include:

- Numbers that end in 0 have 10 as a factor. These numbers are multiples of 10.
- Numbers that end in 0 or 5 have 5 as a factor. These numbers are multiples of 5.
- Even numbers have 2 as a factor. These numbers are multiples of 2.
- Numbers that can be halved twice have 4 as a factor. These numbers are multiples of 4.

What the TEACHER does:

- Engage students in activities in which they use models such as arrays to find all of the factor pairs of a given number. Begin with a smaller range of numbers (1 to 20) and extend the range as students show understanding.
- Use games such as the factor game (http://illuminations.nctm.org/Activity.aspx?id=4134) to provide opportunities for students to find factors.
- As students make lists of factors, provide opportunities for them to discuss patterns.
- Use a variety of activities for students to explore finding multiples by skip counting and relate multiples to the products of a number.
- Have students connect their work with factors to identifying prime numbers (numbers with exactly two factors) and composite numbers (numbers with more than two factors). The multiples of a prime number are all composite numbers.
- Develop mathematical vocabulary including *factor*, *factor pair*, *and multiple*, *odd*, *even*, *prime*, and *composite*. Add these terms to the class mathematics word wall.

What the STUDENTS do:

- Students draw upon and extend their work with multiplication and division facts to determine the factors of a given number through a variety of activities.
- Discuss patterns they discover as they factor a number. (For example, all even numbers have 2 as a factor. Numbers that end in 0 or 5 have 5 as a factor.)
- List multiples of a given number using skip counting and other strategies.
- Identify and describe prime numbers as numbers that have exactly two factors.
- Identify and describe composite numbers as numbers that have more than two factors.

Addressing Student Misconceptions and Common Errors

Students often confuse the terms *factor* and *multiple*. Emphasizing the term *factor* as one of the numbers multiplied to get a product throughout all of the work with multiplication, and expecting students to use that term, should help avoid confusion. Telling students they multiply to get a *multiple* or defining *multiples* of a number as products of the number is also helpful. The more experience students have with these terms, the more accurate they will become when using them.

When listing multiples of a number, students may forget to include the number itself. Reminding students that multiples are the products of a number leads to a discussion of why a number is a factor and a multiple of itself, which is a result of the identity element of multiplication ($a \times 1 = a$).

Students may become confused about whether 1 is a prime or composite number, when actually it is neither prime nor composite because it has only one factor, itself. Developing precise definitions should help to eliminate this misconception.

Notes

Operations and Algebraic Thinking
4.OA.C*

Cluster C

Generate and analyze patterns.

STANDARD 5

4.OA.C.5: Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

*Additional cluster

Operations and Algebraic Thinking 4.OA.C

Cluster C: Generate and analyze patterns.
Grade 4 Overview

Finding, extending, generating, and describing patterns support developing conceptual understanding for all whole-number operations. Finding patterns is also an important strategy for solving problems. Students should have opportunities to extend and describe both physical patterns and numerical patterns.

Standards for Mathematical Practice
SFMP 1. Make sense of problems and persevere in solving them.
SFMP 2. Use quantitative reasoning.
SFMP 3. Construct viable arguments and critique the reasoning of others.
SFMP 4. Model with mathematics.
SFMP 5. Use appropriate tools strategically.
SFMP 7. Look for and make use of structure.
SFMP 8. Look for and express regularity in repeated reasoning.

Students use problems as a context for finding and extending patterns. They reason about similarities and generate rules to describe numerical and geometric patterns. Students use models and tools to describe patterns they find in problems, in numbers, and in geometric figures and to extend these patterns to other situations. They develop lists of numbers given a rule and then describe any patterns in the list using appropriate vocabulary.

In finding patterns, students are developing a deeper understanding of the structure of all four operations and begin to make generalizations by constructing rules for their patterns.

Related Content Standards

3.OA.D.9 5.OA.B.3 5.NBT.A.2

Notes

STANDARD 5 (4.OA.C.5)

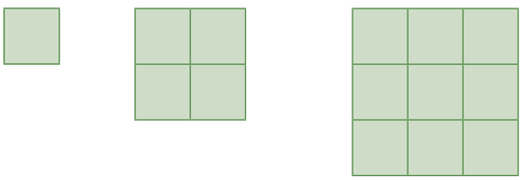
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

Patterns that involve numbers or symbols can be repeating patterns or growing patterns. A repeating pattern is a cyclical repetition of an identifiable core. A linear growing pattern is a pattern that increases or decreases by a constant difference. Patterns can be shown by numeric or by geometric representations.

Number Pattern

Start with the number 4, add 5. 4, 9, 14, 19, 24, 29, . . . The numbers alternate with 4 and 9 in the ones place, which is an example of a repeating pattern. The number sequence increases by 5 which is a growing pattern.

Geometric Pattern



1 4 9

The shapes are all squares.

Each shape has one more tile in each row and column.

The numbers of tiles are multiples of factor pairs with the same factor.

$1 \times 1 = 1$ $2 \times 2 = 4$ $3 \times 3 = 9$

Students need a variety of opportunities in their regular mathematics work to create, extend, and describe patterns. Numerical patterns reinforce mastery of basic facts and understanding operations.

Given a geometric pattern or a numerical rule, students should extend the pattern and describe features of the pattern. They should have opportunities to describe what is happening with the pattern, but they do not need to generalize a particular rule. Describing physical patterns and connecting them to quantity supports recognition of more sophisticated patterns. Building patterns from problem situations and making an organized list are two invaluable problem solving strategies.

Finding a pattern is also an important strategy students can use to solve problems.

Example:

Anna has 5 pennies in her piggy bank. Each day she adds 6 more pennies. How many pennies will Anna have after 5 days?

Day	Pennies Added (equation)	Total Number of Pennies
0	$0 + 5$	5
1	$1 \times 6 + 5$	11
2	$2 \times 6 + 5$	17
3	$3 \times 6 + 5$	23
4	$4 \times 6 + 5$	29
5	$5 \times 6 + 5$	35

Student discuss what they notice and how they can continue the pattern for more days.

What the TEACHER does:

- Provide a variety of problems and models to give students opportunities to recognize, extend, and describe patterns.
- Help students to use a variety of strategies to organize their work so that patterns are more apparent. For example, making a list, extending and describing shapes, making a model will help students to recognize and describe patterns they see.
- Facilitate student discussions about patterns they find so that students become comfortable describing and writing their ideas using words and numbers.

What the STUDENTS do:

- Look for patterns in all of their mathematical work.
- Extend and describe patterns they find.
- When appropriate, make generalizations about patterns.

Addressing Student Misconceptions and Common Errors

Some students will have difficulty recognizing and extending patterns. Others may struggle to describe the pattern. It is important for students to have many experiences with patterns and to recognize those patterns. The more patterns are explored and related to mathematical concepts, the better students will become in recognizing and describing patterns.

Some students may "recognize" a pattern when there is no pattern. Asking students to describe and continue the pattern will help them to differentiate real patterns from non-patterns.

Notes

Sample PLANNING PAGE

4.OA.B

Operations and Algebraic Thinking

Cluster B: Gain familiarity with factors and multiples.

Standard: 4.OA.B.4. *Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.*

Standards for Mathematical Practice:

SFMP 1: Make sense of problems and persevere in solving them.
Students solve this problem to identify factors of given numbers.

SFMP 4: Model with mathematics.
Using a variety of rectangular arrays will help students find factors.

SFMP 6: Attend to precision.
Students work to find all of the possible solutions and use the terms *dimensions* and *factors* in relationship to their models.

SFMP 7: Look for and make use of structure.
Students use physical models to see the structure of multiplication and in a later lesson connect their findings to identifying prime and composite numbers.

Goal:

Students use rectangular arrays to solve a problem and connect their findings to identifying the factors of a given number.

Planning:

Materials: Square tiles, grip paper, problem page, recording sheet

Sample Activity:

- Introduce students to making rectangular arrays from a given number of square tiles.
- Introduce the term *dimensions*.
- Present students with the problem and allow them time to build all the arrays they can using the tiles, draw the arrays on grid paper, and then list the dimensions of the rectangles on the recording sheet.
- Connect the dimensions they have listed to the factors of the number. Then list the factors from least to greatest.
- Discussion follows on the number of factors and the number of boxes and how they are connected.

Notes

Questions/Prompts:

Some students may see the connection between the dimensions of the boxes and the factors and not want to use the materials. Encourage them to use the materials so they can see other patterns in their work and find the "non-basic fact" factors. (For example, 36 has factors of 3 and 12 and 2 and 18.)

Following student work time, be sure to take time to discuss patterns students have found. Questions similar to the following will facilitate the conversation:

- What did you notice about the even numbers?
- Was the same thing true for all of the odd numbers?
- Look at 4, 9, and 16. What do you notice about the number of boxes you found?
- What is different about the number 1?
- What other patterns did you notice?

Differentiating Instruction:

Struggling Students: Struggling students may not have time to work all the way to 40. However, you may find that they enjoy the activity and may want to continue on their own time. Make adjustments accordingly.

It is important that struggling students—especially those who have not mastered their facts—use the concrete materials to complete the task. They also may need support in finding all of the possible rectangles. Asking questions such as "*Can you make a box that is a 4 by something?*" will encourage them to keep trying.

Extension: Check to be sure students who complete the activity early have found *all* of the possible boxes. They can work beyond 40 and may enjoy the challenge of working with larger numbers.

Notes

Sample PLANNING PAGE

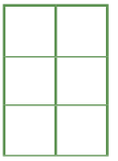
4.OA.B

Yummy Doggy Treats are square in shape. They are sold in rectangular boxes with one layer of treats in a box.

A box of six treats could be any of the following shapes.

1 by 6

6 by 1



____ by ____

____ by ____

The numbers under each box show the *dimensions* of the box. The first box on the left is 1 treat wide and 6 treats high or 1 by 6. The second box is 6 treats wide and 1 treat high. Write the dimensions of the other two boxes.

Our class has been hired to find all of the possible boxes that could be made for any number of Yummy treats from 1 to 40. Work in your groups to complete the table of treats. Look for patterns as you complete the table.

Notes

Table of Treats

Number of Treats	Dimensions of Boxes	Number of Boxes	Factors
6	1 × 6, 6 × 1, 2 × 3, 3 × 2	4	1, 2, 3, 6

PLANNING PAGE

4.OA.A

Operations and Algebraic Thinking
Cluster A: Use the four operations with whole numbers to solve problems.

Standard:

Standards for Mathematical Practice:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

PLANNING PAGE

4.OA.B

Operations and Algebraic Thinking
Cluster B: Gain familiarity with factors and multiples.

Standard:

Standards for Mathematical Practice:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

PLANNING PAGE

4.OA.C

Operations and Algebraic Thinking
Cluster C: Generate and analyze patterns.

Standard:

Standards for Mathematical Practice:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension: